A GENERALIZED PROPORTIONATE-TYPE NORMALIZED SUBBAND ADAPTIVE FILTER

Kuan-Lin Chen, Ching-Hua Lee, Bhaskar D. Rao, and Harinath Garudadri
Department of Electrical and Computer Engineering, University of California, San Diego

Contact Information:
Department of Electrical and Computer Engineering
University of California, San Diego
9500 Gilman Drive #0436, La Jolla, CA 92093

Email: kuc029@ucsd.edu

UC San Diego
JACOBS SCHOOL OF ENGINEERING
Electrical and Computer Engineering

QUALCOMM INSTITUTE

Abstract

We show that a new design criterion, i.e., the least squares on subband errors regularized by a weighted norm, can be used to generalize the proportionate-type normalized subband adaptive filtering (PtNSAF) framework. The new criterion directly penalizes subband errors and includes a sparsity penalty term which is minimized using the damped regularized Newton's method. The impact of the proposed generalized PtNSAF (GPtNSAF) is studied for the system identification problem via computer simulations. Specifically, we study the effects of using different numbers of subbands and various sparsity penalty terms for quasi-sparse, sparse, and dispersive systems.

1 The Least Squares on Subband Errors Regularized by a Weighted Norm

Instead of minimizing the fullband squared error [1], we minimize the sum of the squared error in each subband with a sparsity penalty term. We propose the following cost function:

$$J(\mathbf{s}) = \sum_{i=1}^{M} |e_i(n)|^2 + \tau \|\mathbf{s}\|^2_{\mathbf{W}^{-1}(n)} \quad (1)$$

where $e_i(n) = \mathbf{h}_i^T \mathbf{e}(n) = \mathbf{h}_i^T \left[\mathbf{d}(n) - \mathbf{U}^T(n)\mathbf{s}\right]$ is the i-th subband error and $\mathbf{s} \in \mathbb{R}^L$ is the coefficients of the adaptive filter. The regularization term is designed to expedite the system identification process by introducing a weighted norm for filter taps. We use the $\mathbf{W}(n)$ suggested in [1, 2], i.e., $w_i(n) = \left(|s_i(n)| + c\right)^{2-p}$, $p \in [1.0, 2.0]$, $c > 0, \forall i$ for promoting different degrees of sparsity.

2 Deriving GPtNSAF

To proceed, we perform the affine scaling transform (AST) [3] on the optimization variable $\mathbf{s}$:

$$\mathbf{q} = \mathbf{W}^{-\frac{1}{2}}(n)\mathbf{s}. \quad (2)$$

Applying (2) into (1) , the equivalent optimization problem $\min_{\mathbf{q}} J(\mathbf{q}) = \sum_{i=1}^{M} |e_i(n)|^2 + \tau \|\mathbf{q}\|_2^2$ in $\mathbf{q}$ domain can be easily solved. We define the *a posteriori* AST variable at time n as $\mathbf{q}(n|n) \triangleq \mathbf{W}^{-\frac{1}{2}}(n)\mathbf{s}(n)$ and the *a priori* AST variable as $\mathbf{q}(n+1|n) \triangleq \mathbf{W}^{-\frac{1}{2}}(n)\mathbf{s}(n+1)$. Now, we consider the damped regularized Newton's method for the update rule on minimizing $J(\mathbf{q})$, i.e., $\mathbf{q}(n+1|n) = \mathbf{q}(n|n) - \mu\left[\nabla^2_{\mathbf{q}} J\left(\mathbf{q}(n|n)\right) + 2\delta\mathbf{I}\right]^{-1} \nabla_{\mathbf{q}} J\left(\mathbf{q}(n|n)\right)$ where $\mu > 0$ is the learning rate or the step size for adaptation and $\delta > 0$ is a regularization parameter. The gradient of $J(\mathbf{q})$ is given by

$$\nabla_{\mathbf{q}} J(\mathbf{q}(n|n)) = -2\mathbf{W}^{\frac{1}{2}}(n)\mathbf{U}_b(n)\mathbf{e}_b(n) + 2\tau\mathbf{q}(n|n). \quad (3)$$

Next, the Hessian is given by

$$\nabla^2_{\mathbf{q}} J(\mathbf{q}(n|n)) = 2\mathbf{W}^{\frac{1}{2}}(n)\mathbf{U}_b(n)\mathbf{U}_b^T(n)\mathbf{W}^{\frac{1}{2}}(n) + 2\tau\mathbf{I}. \quad (4)$$

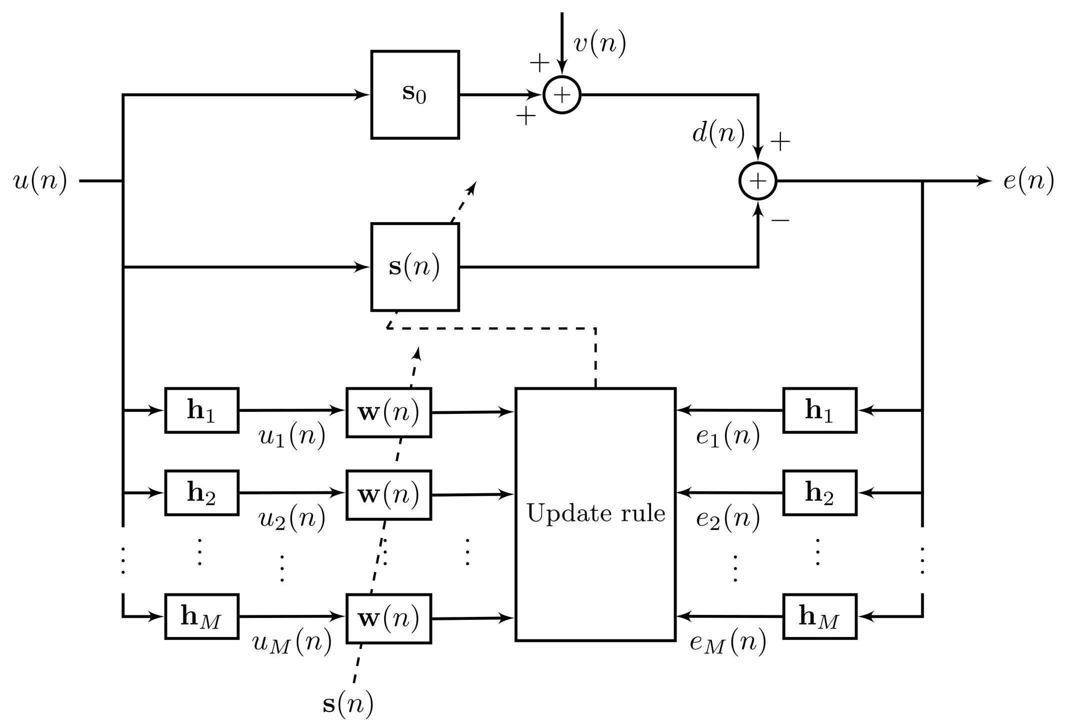

Figure 1: Block diagram of the GPtNSAF.

Therefore, the update rule on $\mathbf{q}$ domain is given by

$$\mathbf{q}(n+1|n) = \left(\mathbf{I} - \frac{\mu\tau}{\delta+\tau}\left[\mathbf{I} - \mathbf{\Psi}(n)\right]\right)\mathbf{q}(n|n) + \mu\mathbf{W}^{\frac{1}{2}}(n)\mathbf{U}_b(n)\mathbf{\Phi}(n)\mathbf{e}_b(n) \quad (5)$$

where we have applied the Woodbury matrix identity to avoid large matrix inversion (L-by-L) in the damped regularized Newton's method and $\mathbf{\Psi}(n) = \mathbf{W}^{\frac{1}{2}}(n)\mathbf{U}_b(n)\mathbf{\Phi}(n)\mathbf{U}_b^T(n)\mathbf{W}^{\frac{1}{2}}(n)$. Notice that the inverse of the regularized weighted subband correlation matrix, i.e.,

$$\mathbf{\Phi}(n) = \left[(\delta+\tau)\mathbf{I}_M + \mathbf{U}_b^T(n)\mathbf{W}(n)\mathbf{U}_b(n)\right]^{-1} \quad (6)$$

is a very small matrix inversion which only has M-by-M since we have $L \gg M$ in most cases. Then, by utilizing (2) in (5) to convert $\mathbf{q}$ back to the $\mathbf{s}$ domain, we have

$$\mathbf{s}(n+1) = \left(\mathbf{I} - \frac{\mu\tau}{\delta+\tau}\left[\mathbf{I} - \mathbf{\Psi}(n)\right]\right)\mathbf{s}(n) + \mu\mathbf{W}(n)\mathbf{U}_b(n)\mathbf{\Phi}(n)\mathbf{e}_b(n). \quad (7)$$

Finally, setting $\tau \to 0^+$ leads to the update rule for the GPtNSAF: $\mathbf{s}(n+1) = \mathbf{s}(n) + \mu\mathbf{g}(n)$ where

$$\mathbf{g}(n) = \mathbf{W}(n)\mathbf{U}_b(n)\left[\delta\mathbf{I}_M + \mathbf{U}_b^T(n)\mathbf{W}(n)\mathbf{U}_b(n)\right]^{-1}\mathbf{e}_b(n). \quad (8)$$

We have used $\mathbf{U}_b(n) = \mathbf{U}(n)\mathbf{H}$ and $\mathbf{e}_b(n) = \mathbf{H}^T\mathbf{e}(n) \in \mathbb{R}^M$ where $\mathbf{H} \in \mathbb{R}^{N\times M}$ is an M-channel analysis filter bank matrix.

3 Special Cases of the GPtNSAF

- **PtNSAF**: By selecting $\mathbf{H}$ as the set of eigenvectors for the weighted correlation matrix $\mathbf{U}^T(n)\mathbf{W}(n)\mathbf{U}(n)$, we have the PtNSAF: $\mathbf{g}(n) = \sum_{i=1}^{M} \frac{e_i(n)}{\mathbf{u}_i^T(n)\mathbf{W}(n)\mathbf{u}_i(n)+\delta}\mathbf{W}(n)\mathbf{u}_i(n)$.
- **NSAF**: Based on PtNSAF, setting $\mathbf{W}(n) = \mathbf{I}$ gives the NSAF: $\mathbf{g}(n) = \sum_{i=1}^{M} \frac{e_i(n)}{\mathbf{u}_i^T(n)\mathbf{u}_i(n)+\delta}\mathbf{u}_i(n)$.
- **PtNLMS**: Setting $M = N = 1$ yields $\mathbf{H} = 1 \in \mathbb{R}$, thus we get the PtNLMS: $\mathbf{g}(n) = \frac{e(n)}{\mathbf{u}^T(n)\mathbf{W}(n)\mathbf{u}(n)+\delta}\mathbf{W}(n)\mathbf{u}(n)$.
- **NLMS**: Based on PtNLMS, setting $\mathbf{W}(n) = \mathbf{I}$ gives the NLMS: $\mathbf{g}(n) = \frac{e(n)}{\mathbf{u}^T(n)\mathbf{u}(n)+\delta}\mathbf{u}(n)$.

4 Simulation Results

The input signal is a first order autoregressive (AR) process. The analysis bank $\mathbf{H}$ is a cosine-modulated pseudo-quadrature mirror filter (QMF) bank. The MSE curves were obtained as the ensemble average over 1000 Monte Carlo runs and normalized to start from 0 dB. For all MSE simulations, we used $\mu = \frac{0.2}{M}$.

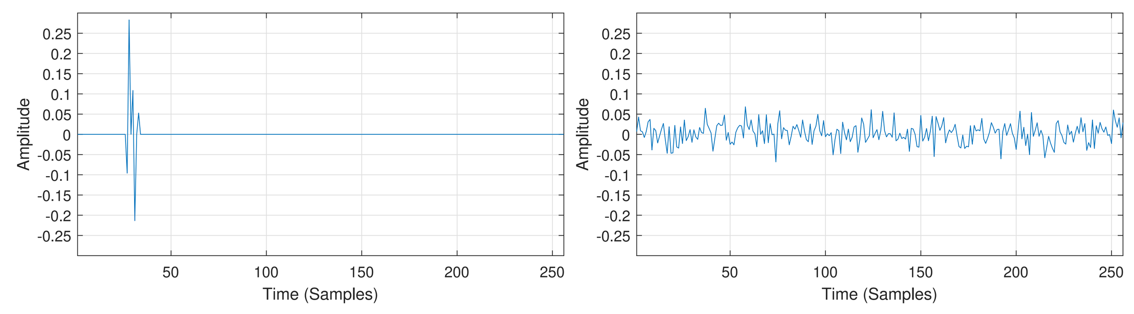

Figure 2: Different impulse responses (IRs), namely, target systems of length $L = 256$ with different degrees of sparsity. Left: sparse; right: dispersive.

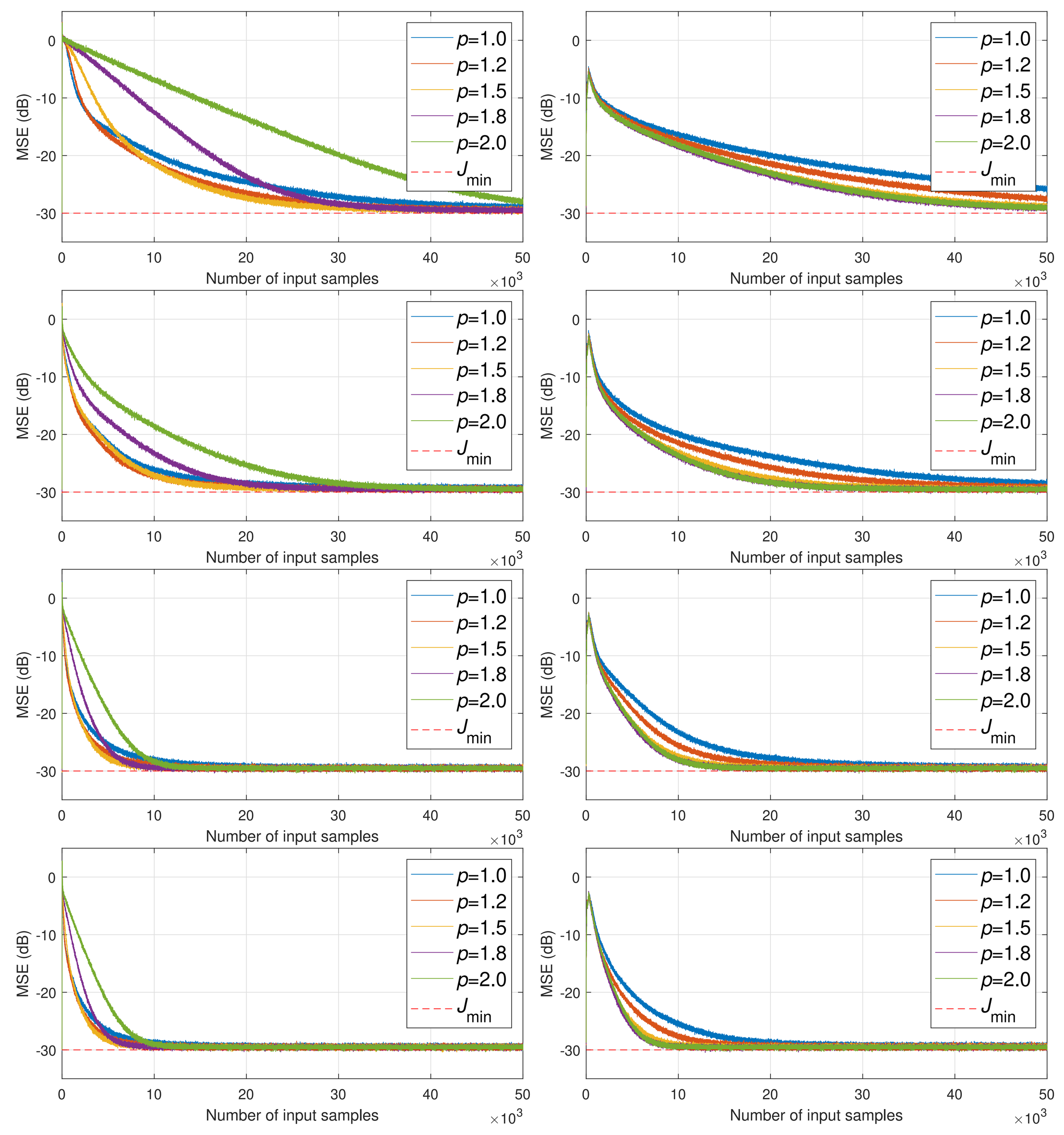

Figure 3: From the top to the bottom row: $M = 1, 2, 4, 8$. From left to right: sparse IR to dispersive IR. The best p values on each target system are consistent across different numbers of subbands. Therefore, we suggest $p \in [1.0, 1.2]$ and $p \in [1.8, 2.0]$ for sparse and dispersive target systems, respectively. Notice that the convergence speed is significantly improved for all target systems as the number of subbands increases. The benefits of increasing M and incorporating $\mathbf{W}(n)$ are complementary and additive. The benefits of increasing M and incorporating $\mathbf{W}(n)$ are complementary and additive for fast convergence.

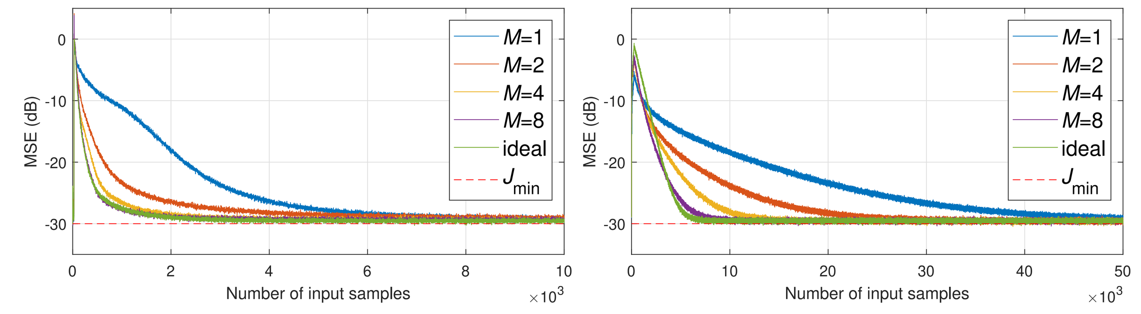

Figure 4: We use the suggested p values for $M = 1, 2, 4, 8$ (left: $p = 1.2$; right: $p = 1.8$). By increasing the number of subbands, the MSE curves with colored input signal approach the ideal case, i.e., the GPtNSAF with $M = 1$ using white input signal. Note that this ideal case is equivalent to the propotionate-type NLMS with white input.

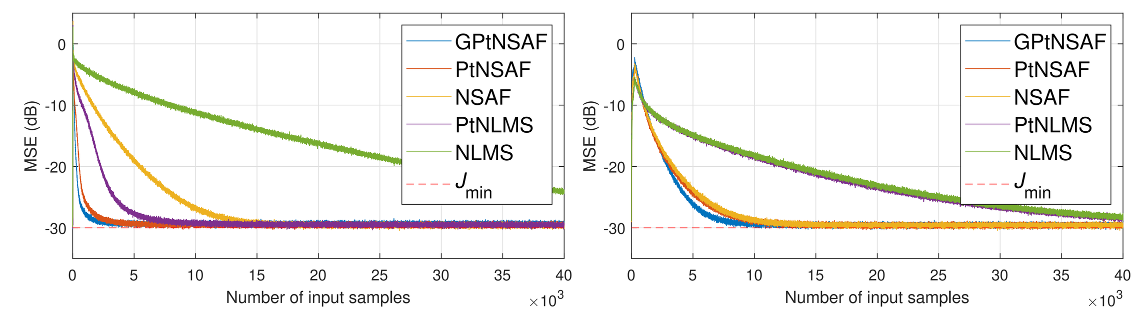

Figure 5: PtNSAF approximates GPtNSAF under different degrees of sparsity since the magnitude responses of the analysis filters do not significantly overlap (diagonal assumption). To sum up, GPtNSAF yields the best convergence speed than the others as we expected under all cases.

5 Conclusion

A generalized PtNSAF is proposed to further improve the convergence speed based on directly minimizing subband errors with a sparsity penalty term. Different adaptive filters including the PtNSAF, NSAF, PtNLMS, and NLMS can be obtained by choosing the corresponding hyperparameters of GPtNSAF. The benefits of increasing the number of subbands and promoting different degrees of sparsity of the estimated filter coefficients are compared under various environments. The simulation results show that the proposed GPtNSAF is suitable for identifying quasi-sparse, sparse, and dispersive systems under colored excitation. More importantly, the two aspects of using M and p provide complementary and additive benefits for speeding up convergence.

6 Acknowledgements

The authors would like to thank Prof. B. D. Rao for fruitful discussions. This work was supported by NIH/NIDCD grants R01DC015436 and R33DC015046.

References

[1] C.-H. Lee, B. D. Rao, and H. Garudadri, "Sparsity promoting LMS for adaptive feedback cancellation," in *2017 25th European Signal Processing Conference (EUSIPCO)*, Aug 2017, pp. 226–230.

[2] B. D. Rao and B. Song, "Adaptive filtering algorithms for promoting sparsity," in *2003 IEEE International Conference on Acoustics, Speech, and Signal Processing, 2003. Proceedings. (ICASSP '03).*, vol. 6, April 2003, pp. VI–361.

[3] B. D. Rao and K. Kreutz-Delgado, "An affine scaling methodology for best basis selection," *IEEE Transactions on Signal Processing*, vol. 47, no. 1, pp. 187–200, Jan 1999.